

Many flavor QCD as exploration of the walking behavior with the approximate IR fixed point

Yasumichi Aoki^a, Tatsumi Aoyama^a, Masafumi Kurachi^a, Toshihide Maskawa^a, Kei-ichi Nagai^{*†a}, Hiroshi Ohki^a, Akihiro Shibata^b, Koichi Yamawaki^a and Takeshi Yamazaki^a

LatKMI Collaboration

^a*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*

^b*Computing Research Center, High Energy Accelerator Research Organization (KEK), Tsukuba, 305-0801, Japan*

We present the first report of the LatKMI collaboration on the the lattice QCD simulation performed at the KMI computer, “ ϕ ”, for the cases of 4 flavors and 8 flavors, the latter being expected to be a candidate for the walking technicolor having an approximate scale invariance near the infrared fixed point. The simulation was carried out based on the highly improved staggered quark (HISQ) action. In this proceedings, we report preliminary results on the spectrum, analyzed through the chiral perturbation theory and the finite-size hyperscaling. We observe qualitatively different behavior of the 8-flavor case in contrast to the 4-flavor case which shows clear indication of the hadronic phase as in the usual QCD.

*The XXIX International Symposium on Lattice Field Theory - Lattice 2011
July 10-16, 2011
Squaw Valley, Lake Tahoe, California*

^{*}Speaker.

[†]E-mail: keiichi.nagai@kmi.nagoya-u.ac.jp

1. Introduction

The origin of mass is the most urgent issue of the particle physics today. One of the candidates for the theory beyond the Standard Model towards that problem is the walking technicolor which is the strongly coupled gauge theory having a large anomalous dimension $\gamma_m \simeq 1$ and approximate scale invariance due to the almost non-running (walking) coupling [1, 2]. The walking behavior is in fact realized in the QCD with large number of (massless) flavors N_f which possesses Caswell-Banks-Zaks infrared fixed point (IRFP) [3] for the value larger than $N_f \simeq 8.0$ in the two-loop beta function. The exact IRFP would be washed out by the dynamical generation of a quark mass m in the very infrared region $\mu < m$ for $N_f < N_f^{cr}$, N_f^{cr} being the critical number. However, for N_f very close to N_f^{cr} , m could be much smaller than the intrinsic scale Λ ($\gg m$), an analogue of Λ_{QCD} , beyond which the coupling runs as the asymptotically free theory, so that the coupling remains almost non-running for the wide infrared region $m < \mu < \Lambda$ as a remnant of the would-be IRFP. The case $N > N_f^{cr}$ is called conformal window, although conformality is broken in the ultraviolet asymptotically free region beyond Λ . The critical number N_f^{cr} was estimated as $N_f^{cr} \simeq 11.9$ [4] by comparing the two-loop IRFP value with the critical coupling of the ladder Schwinger-Dyson equation analysis [5].

Although the above results from the two-loop and ladder approximation are very suggestive, the relevant dynamics is obviously of non-perturbative nature, we would need fully non-perturbative studies. Among others the lattice simulations developed in the lattice QCD would be the most powerful tool for that purpose. Our group, LatKMI Collaboration, was organized for such studies on walking technicolor as a candidate for the theory beyond the Standard Model. The immediate issues are: What is the critical number N_f^{cr} ? What is the signatures of the walking theory expected to be slightly smaller than N_f^{cr} ? The above two-loop and ladder studies suggest that the walking theory if existed would be in between $N_f = 8$ and $N_f = 12$. The $N_f = 8$ in particular is interesting from the model-building point of view: The typical technicolor model [6] is the so-called one-family model (Farhi-Susskind model) which has a one-family of the colored techni-fermions (techni-quarks) and the uncolored one (techni-leptons) corresponding to the each family of the SM quarks and leptons. It can embed the technicolor gauge and the gauged three generations of the SM fermions into a single gauge group (Extended Technicolor) and thus is the most straightforward way to accommodate the techni-fermions and the SM fermions into a simple scheme. Thus if the $N_f = 8$ turns out to be a walking theory, it would be a great message for the phenomenology to be tested by the on-going LHC.

To date, some groups [7, 8, 9, 10, 11, 12] carried out lattice studies on 8 flavors; Ref. [7] computed the running coupling constant in $N_f = 8$ by the Schrödinger functional method in staggered fermion case and reported that $N_f = 8$ is in the chiral broken phase. Refs. [8, 9, 10, 11, 12] investigated the hadron spectrum with the standard Wilson fermions [8], the Stout improved staggered fermions [10], the Asqtad improved staggered fermions [11] and the naïve staggered fermions [9, 12]. The Ref. [8] concluded the $N_f = 8$ is in the conformal window, but other groups concluded that the $N_f = 8$ resides on the chiral broken phase. Even if $N_f = 8$ is in the chiral broken phase, it is not clear whether the behavior of this system is *QCD-like* or the walking with the large anomalous mass dimension. Nobody has investigated the possibility that $N_f = 8$ is in the walking.

We simulate 8-flavor QCD with alternative lattice fermion, HISQ, in which the flavor symme-

try in the staggered fermion is improved and, of course, it is expected that the behavior towards the continuum limit is improved. We show the preliminary result of the hadron spectrum and analyzed the data based on the hyperscaling [13] as well as the chiral perturbation theory (ChPT). From the hyperscaling analysis, we derive the anomalous mass dimension γ_m . We observe qualitatively different behavior of the 8-flavor case in contrast to the 4-flavor case which shows clear indication of the hadronic phase as in the usual QCD.

2. Simulation

In our simulation, we use the tree level Symanzik gauge action and the highly improved staggered quark (HISQ) action without the tadpole improvement and the mass correction in the Naik term. See Ref. [14] for the detail of the HISQ action. We use the MILC code [15] with modifications to simulate $N_f = 4n$ HISQ by using the standard Hybrid Monte-Carlo (HMC) algorithm. We computed the hadron spectrum as the global survey in the parameter region and we obtained M_π , M_ρ , f_π and $\langle \bar{\psi}\psi \rangle$ as the basic observable.

The simulation for $N_f = 4$ is carried out at $\beta (= 6/g^2) = 3.5, 3.6, 3.7$ and 3.8 for various quark masses on $12^3 \times 16$ and $16^3 \times 24$. We took over 1000 trajectories on the small lattice and about 600 trajectories on the large lattice in $N_f = 4$ case. The simulation for $N_f = 8$ is carried out at $\beta (= 6/g^2) = 3.6, 3.7$ and 3.8 for various quark masses on $12^3 \times 32$ and $24^3 \times 32$, and for $m_f = 0.02$ and 0.04 on $30^3 \times 40$ at $\beta = 3.7$. We took about 1000 trajectories on $12^3 \times 32$, between 200 and 400 trajectories on $24^3 \times 32$ and 600 trajectories on $30^3 \times 40$.

See Ref. [16] for our simulation in $N_f = 12$ and 16 .

3. Spectrum

In this section, we show preliminary results of ChPT analysis and the finite-size hyperscaling analysis in 4- and 8-flavor cases.

3.1 $N_f = 4$

The result of $N_f = 4$ is shown in Fig. 1. The data on $16^3 \times 24$ at $\beta = 3.7$, the pion mass squared, the decay constant and the chiral condensate are plotted on the panel from the left to the right respectively. M_π^2 is proportional to m_f . f_π and $\langle \bar{\psi}\psi \rangle$ have the non zero value in the chiral limit. Thus, the $N_f = 4$ QCD has the property of the chiral broken phase and this is regarded as the signal of the chiral broken phase in the dynamical case of lattice QCD.

3.2 $N_f = 8$

In this subsection, we analyze $N_f = 8$ system by ChPT and the finite-size hyperscaling relation. In Fig. 2, as the typical example of the raw data, we plotted the preliminary data of M_π and M_ρ as a function of the quark mass m_f on $12^3 \times 32$ and $24^3 \times 32$. It is shown that M_π and M_ρ are similar behavior, that is, the plateau appears in $m_f \lesssim 0.06$ on small lattice ($12^3 \times 32$) at all β s and in $m_f \lesssim 0.02$ on large lattice ($24^3 \times 32$) at $\beta = 3.8$. In the following, we attempt ChPT and the finite-size hyperscaling analysis by using these data.

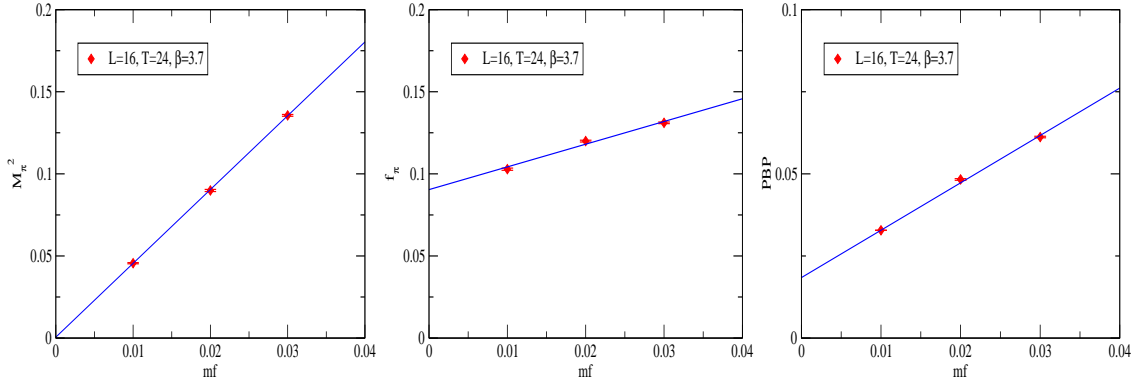


Figure 1: In $N_f = 4$ $SU(3)$ gauge theory on $16^3 \times 24$ at $\beta = 3.7$; **Left:** M_π^2 as functions of m_f , **Center:** f_π as functions of m_f , **Right:** $\langle \bar{\psi}\psi \rangle$ as functions of m_f . In all panels, the blue-solid line is the linear fit.

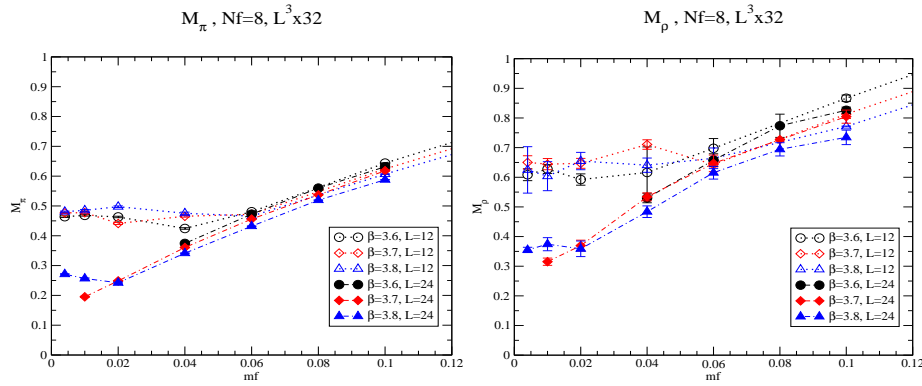


Figure 2: In $N_f = 8$ $SU(3)$ gauge theory on $12^3 \times 32$ and $24^3 \times 32$ at $\beta = 3.6, 3.7$ and 3.8 ; **Left panel:** M_π as functions of m_f , **Right panel:** M_ρ as functions of m_f .

3.2.1 ChPT analysis

We analyze the $N_f = 8$ data by ChPT. Here, we pick up the data at $\beta = 3.7$, which has the $30^3 \times 40$ volume data. Fig. 3 shows M_π^2 , f_π and $\langle \bar{\psi}\psi \rangle$. For each m_f , the largest volume data is used for the quadratic-fit corresponding to ChPT analysis. The data corresponding to the plateau is not included into the fit.

As the fit result, M_π^2 in the chiral limit is consistent with zero. This is fairly independent of the fit range. The quadratic-fit of f_π in the chiral limit indicates non-zero value. These properties are consistent with the chiral broken phase. On the other hand, the condensate is not inconsistent with zero in the chiral limit, in contrast to $N_f = 4$ case (compare Fig. 3 with Fig. 1).

The definite conclusion cannot be given here because $\chi^2/d.o.f. \sim O(10)$ in ChPT fit and because it is difficult to take the infinite volume limit. Even if $N_f = 8$ is in the broken phase, there is a possibility that the walking behavior as a remnant of the conformality can be observed in the form of hyperscaling relation.

3.2.2 Finite-size hyperscaling analysis

If the system is in the conformal window, physical quantities, M_H , are described by the finite-

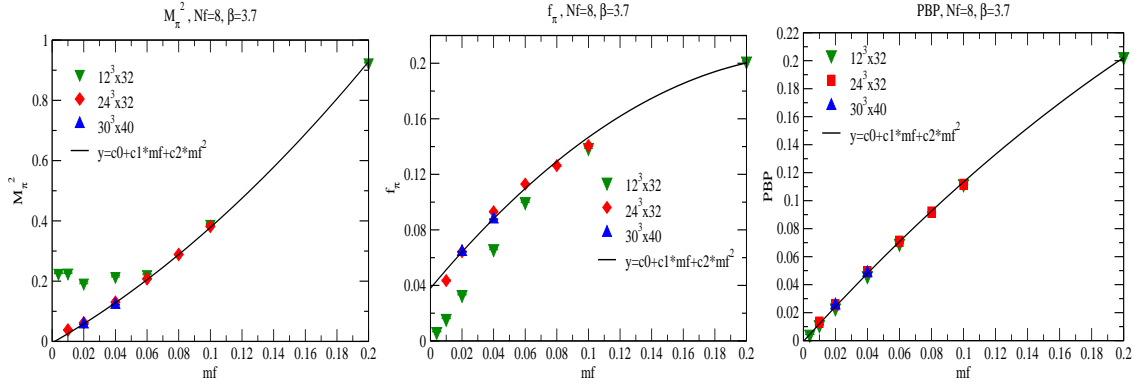


Figure 3: In $SU(3)$ gauge theory with 8-flavors on $12^3 \times 32$, $24^3 \times 32$ and $30^3 \times 40$ at $\beta = 3.7$; **Left:** M_π^2 as functions of m_f . $\chi^2/d.o.f. = 42$. **Center:** f_π . $\chi^2/d.o.f. = 105$. **Right:** $\langle \bar{\psi}\psi \rangle$. $\chi^2/d.o.f. = 154$; In all panels, the solid line stands for the quadratic-fit.

size hyperscaling relation [13] ; $LM_H = \mathcal{F}(X)$ where $\mathcal{F}(X)$ is unknown function with the scaling variable $X = Lm_f^{\frac{1}{1+\gamma}}$. The γ in this equation is defined as the anomalous mass dimension. We carry out the hyperscaling analysis with our data of $M_H = \{M_\pi, f_\pi, M_\rho\}$ by the following fit function

$$LM_H = c_0 + c_1 Lm_f^{\frac{1}{1+\gamma}}. \quad (3.1)$$

Figs. 4, 5 and 6 are the finite-size hyperscaling result with this fitting of LM_π , Lf_π and LM_ρ respectively. The filled symbol is applied to the hyperscaling fitting. These data on various lattice sizes align at an optimal value of γ . Although the fit quality is more or less the same level as that of the ChPT fit, we extracted the γ -value $\gamma(M_\pi) \sim 0.6$, $\gamma(f_\pi) \sim 1.0$ and $\gamma(M_\rho) \sim 0.8$ at all β , which are not universal. Therefore, our result of 8 flavors does not show the clear signature of the conformal window. Still, this situation in $N_f = 8$ is qualitatively different from the case of $N_f = 4$. Actually we did not find even an alignment of the hyperscaling for each physical quantity in $N_f = 4$.

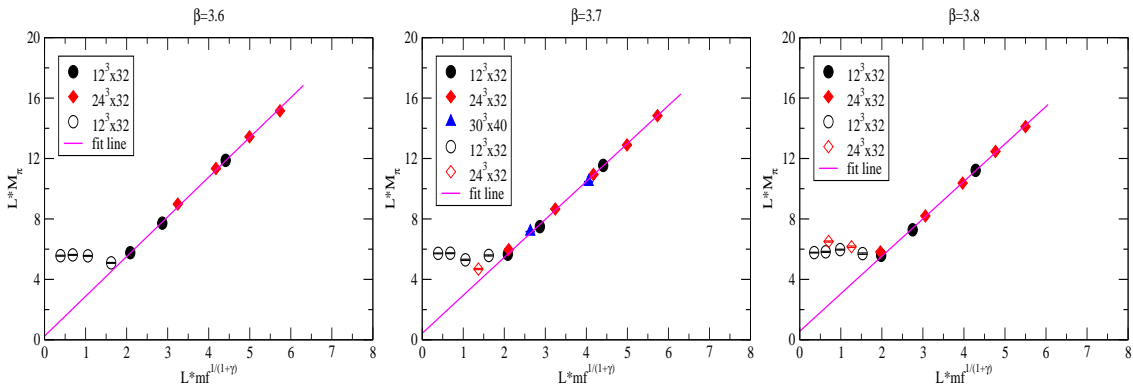


Figure 4: **Left:** The hyperscaling relation of LM_π as functions of $Lm_f^{\frac{1}{1+\gamma}}$ in $N_f = 8$ at $\beta = 3.6$, $\chi^2/d.o.f. = 41.7$. **Center:** at $\beta = 3.7$, $\chi^2/d.o.f. = 39.8$. **Right:** at $\beta = 3.8$, $\chi^2/d.o.f. = 37.3$. At all β , $\gamma(M_\pi) \sim 0.6$. The filled symbol is applied to the hyperscaling fit.

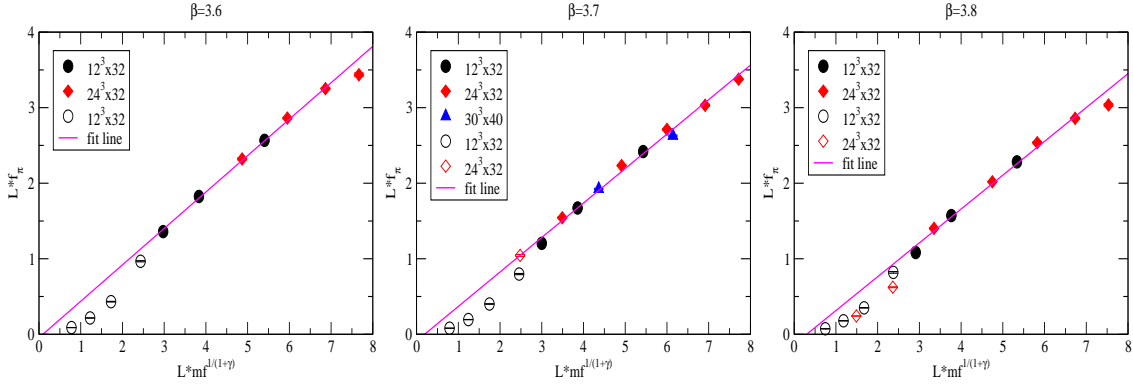


Figure 5: **Left:** The hyperscaling relation of Lf_π as functions of $Lm_f^{1/(1+\gamma)}$ in $N_f = 8$ at $\beta = 3.6$, $\chi^2/d.o.f. = 30.9$. **Center:** at $\beta = 3.7$, $\chi^2/d.o.f. = 57.8$. **Right:** at $\beta = 3.8$, $\chi^2/d.o.f. = 55.1$. At all β , $\gamma(f_\pi) \sim 1.0$.

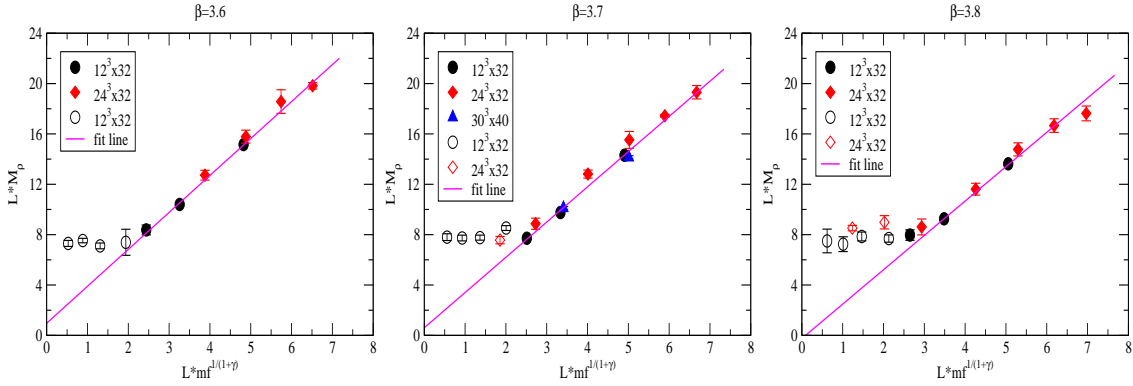


Figure 6: **Left:** The hyperscaling relation of LM_ρ as functions of $Lm_f^{1/(1+\gamma)}$ in $N_f = 8$ at $\beta = 3.6$, $\chi^2/d.o.f. = 1.3$. **Center:** at $\beta = 3.7$, $\chi^2/d.o.f. = 5.7$. **Right:** at $\beta = 3.8$, $\chi^2/d.o.f. = 2.6$. At all β , $\gamma(M_\rho) \sim 0.8$.

4. Summary

We have made simulations of lattice QCD with 4 and 8 flavors by using the HISQ action. We obtained the following result; The $N_f = 4$ QCD is in good agreement with the chiral broken phase. The $N_f = 8$, on the other hand, does not seem to be inconsistent with both ChPT and the finite-size hyperscaling, with $\chi^2/d.o.f.$ not being small for both analyses. We extracted the γ -value from the hyperscaling analysis, $\gamma(M_\pi) \sim 0.6$, $\gamma(f_\pi) \sim 1.0$ and $\gamma(M_\rho) \sim 0.8$ at all β , which are not universal. Therefore, our result of 8 flavors does not show the clear signature of the conformal window. This may be an indication of the walking behavior that appears in the broken phase just below the edge of the conformal window.

We should mention that there are several possible systematic uncertainties not considered in this report; Our data include those with slightly different aspect ratio, the ratio of the temporal length to the spatial length. Our result of the finite-size hyperscaling analysis may suffer from the systematic error due to the different aspect ratio. Furthermore, as pointed out in Ref. [17], there exists the mass correction in the hyperscaling relation for the heavy quark region. Also the linear ansatz adapted in Eq. (3.1) may not be sufficient to fit our data. To improve the situation for better understanding, we will accumulate more data for various fermion masses and β 's on larger lattices,

and carry out detailed analysis using those data.

Acknowledgments

Numerical simulation has been carried out on the supercomputer system “ ϕ ”, which is installed at KMI for the studies beyond the Standard Model. We thank Katsuya Hasebe for the useful discussion. This work is supported by the JSPS Grant-in-Aid for Scientific Research (S) No.22224003, (C) No.23540300 (K.Y.) and (C) No.21540289 (Y.A.), and also by Grants-in-Aid of the Japanese Ministry for Scientific Research on Innovative Areas No. 23105708 (T.Y.).

References

- [1] K. Yamawaki, M. Bando and K. i. Matumoto, Phys. Rev. Lett. **56** (1986) 1335;
- [2] Subsequent similar works without notion of anomalous dimension and scale symmetry were done: T. Akiba and T. Yanagida, Phys. Lett. B **169**, 432 (1986); T. W. Appelquist, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986). See also the earlier work on pure numerical analysis: B. Holdom, Phys. Lett. B **150**, 301 (1985).
- [3] W. E. Caswell, Phys. Rev. Lett. **33** (1974) 244; T. Banks and A. Zaks, Nucl. Phys. B **196** (1982) 189.
- [4] T. Appelquist, J. Terning and L. C. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996)
- [5] T. Maskawa and H. Nakajima, Prog. Theor. Phys. **52**, 1326 (1974); R. Fukuda and T. Kugo, Nucl. Phys. B **117** (1976), 250; V.A. Miransky, Int.J.Mod.Phys.A25:5105-5113,2010.
- [6] See for a review: E. Farhi and L. Susskind, Phys. Rept. **74**, 277 (1981).
- [7] T. Appelquist, G. T. Fleming and E. T. Neil, Phys. Rev. Lett. **100** (2008) 171607 [Erratum-ibid. **102** (2009) 149902] [arXiv:0712.0609 [hep-ph]]; T. Appelquist, G. T. Fleming and E. T. Neil, Phys. Rev. D **79** (2009) 076010 [arXiv:0901.3766 [hep-ph]].
- [8] Y. Iwasaki, K. Kanaya, S. Sakai and T. Yoshie, Phys. Rev. Lett. **69** (1992) 21; Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai and T. Yoshie, Phys. Rev. D **69** (2004) 014507 [arXiv:hep-lat/0309159].
- [9] F. R. Brown, H. Chen, N. H. Christ, Z. Dong, R. D. Mawhinney, W. Schaffer and A. Vaccarino, Phys. Rev. D **46** (1992) 5655 [hep-lat/9206001].
- [10] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, Phys. Lett. B **681** (2009) 353 [arXiv:0907.4562 [hep-lat]].
- [11] A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Lett. B **670** (2008) 41 [arXiv:0804.2905 [hep-lat]].
- [12] X. -Y. Jin and R. D. Mawhinney, PoS LATTICE **2010** (2010) 055 [arXiv:1011.1511 [hep-lat]].
- [13] L. Del Debbio and R. Zwicky, Phys. Rev. D **82** (2010) 014502 [arXiv:1005.2371 [hep-ph]].
- [14] E. Follana *et al.* [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. D **75** (2007) 054502 [arXiv:hep-lat/0610092]; A. Bazavov *et al.* [MILC collaboration], Phys. Rev. D **82** (2010) 074501 [arXiv:1004.0342 [hep-lat]].
- [15] <http://www.physics.utah.edu/~detar/milc/>
- [16] Y. Aoki *et al.* [LatKMI collaboration], PoS(Lattice 2011)053.
- [17] Y. Aoki *et al.* [LatKMI collaboration], arXiv:1201.4157 [hep-lat].